Signals

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Introduction

This is just short reference work for signal theory. Most of it is taken from Lathi's "Linear Systems and Signals," but I find it hard to scan through that book due to all the examples, so most of the important results are summarized neatly here.

Discrete Time Signals

A continuous time signal $x(t)$ can be made discrete by sampling $x(t)$ every T seconds. If we start sampling at $t = 0$, then the points in time, where we sample are nT, where $n \in \mathbb{Z}_0^+$. Therefore, define

$$x[n] \equiv x(nT). \tag{1}$$

Linear systems can be described by difference equations. A difference equation relates various inputs of a system, $x[n]$, to the outputs, $y[n]$. In general, the difference equation is given by

$$\begin{aligned} y[n+N] &+ a_1 y[n+N-1] + ... + a_{N-1} y[n+1] + a_N y[n] \\ &= b_{N-M} x[n+M] + b_{N-M+1} x[n+M-1] + ... + b_{N-1} x[n+1] + b_N x[n]. \end{aligned} \tag{2}$$

We are free to multiply both sides by a constant, which allows us to choose $a_0 = 1$.

Causality says that an output cannot be affected by a future input. In other words, $y[n]$ cannot depend on $x[m]$ with $m > n$. In the difference equation, we then require that $N \geq M$ to ensure this. To be definite,

$$\text{Causality} \implies N \geq M. \tag{3}$$

We define the operator E as that which advances the sequence by one time step, i.e.

$$Ex[n] \equiv x[n+1]. \tag{4}$$

We may then recast the difference equation as

$$\begin{aligned} &(E^N + a_1 E^{N-1} + ... + a_{N-1} E + a_N) y[n] \\ &\quad = (b_{N-M} E^M + b_{N-M+1} E^{M-1} + ... + b_{N-1} E + b_N) x[n]. \end{aligned} \tag{5}$$

If we define two polynomials Q and P as

$$Q[E] \equiv E^N + a_1 E^{N-1} + ... + a_{N-1} E + a_N \tag{6}$$

$$P[E] \equiv b_{N-M}E^M + b_{N-M+1}E^{M-1} + ... + b_{N-1}E + b_N \tag{7}$$

then we may rewrite the difference equation as

$$Q[E]y[n] = P[E]x[n]. \tag{8}$$

The transfer function is defined as

$$H[z] \equiv \frac{P[z]}{Q[z]} = \frac{b_{N-M}z^M + b_{N-M+1}z^{M-1} + ...b_{N-1}z + b_N}{z^N + a_1 z^{N-1} + ...a_{N-1}z + a_N}. \tag{9}$$

Since the causality condition is $N \geq M$, this merely says that the order of the denominator polynomial Q is greater than or equal to the order of the numerator polynomial P.